EWPT dynamics $\longrightarrow$ Bubble Growth & dynamics of plasma

⇒ Towards a complete picture of the dynamics of EWPT and its effects on EWBG and GW

References:
- Steinhardt (82), PRD 25, 2074
- Ignatius, Kajantie, Kurki-Suonio, Laine (hep-ph/9309059)
- Laine (hep-ph/9309242)
- Espinosa, Konstandin, No, Servant (1004.4187)

- Parameters: V_W , $\alpha = \dfrac{\Delta V}{a_+ T^4}$

  $a_+ T^4 \to$ thermal energy of plasma

  $a_+ \to$ # of relativistic degrees of freedom in $\langle\phi\rangle = 0$

  They are independent! (V_W depends on ΔV and on friction η)

- Bubble expansion solutions and ~~the~~ their main features

- Effects of hydrodynamics on EWBG (plasma velocity and temperature around wall) and GW (energy budget)

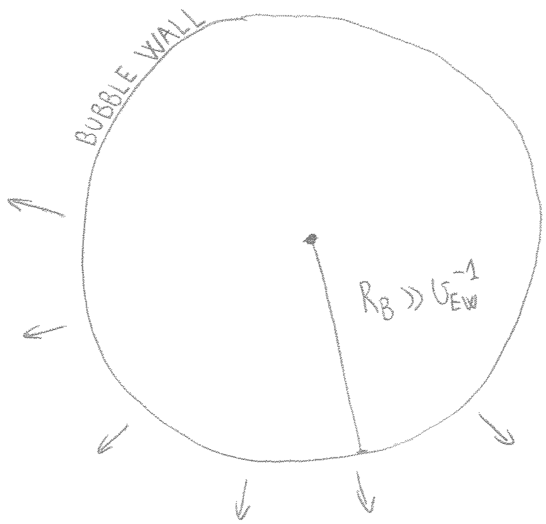
BUBBLE EXPANSION

(See Jonathan's notes!!)

Two regimes:

- Accelerated expansion → Runaway
  - Most of energy of the EWPT into bubble wall
  - $V_W \to 1$ (Good for GW; Bad for EWBG)
- Stationary state ($V_W = Cte$)
  - Most of energy of EWPT into plasma
  - **I WILL FOCUS ON THIS**

We will assume stationary bubble expansion: bubble was created, it started expanding in an accelerated way, eventually friction balanced the pressure difference and lead to a constant V_W.



Impose energy momentum conservation in the system.

$$T^{\phi}_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu}\left[\tfrac{1}{2}\partial_\rho \phi \partial^\rho \phi - V(\phi)\right]$$

$$T_{\mu\nu} = \omega\, u_\mu u_\nu - g_{\mu\nu}\, p$$

ω: enthalpy $\left(T\frac{\partial p}{\partial T}\right)$; u_μ: $(\gamma, \gamma\vec{v})$; p: pressure

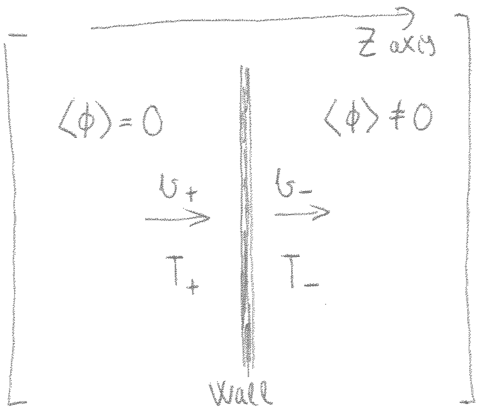
(Simple!) Energy-momentum tensor for a relativistic fluid in <u>local thermal equilibrium</u>

The interactions in the plasma that keep it in equilibrium are much faster than the wall expansion.

1 Energy-momentum conservation across the bubble wall

- Go to bubble wall frame
- Stationary state

$\left.\right\}$ time independence $\Rightarrow \partial_\mu T^{\mu\nu} = 0 \nearrow \partial_z T^{z0} = 0$, $\searrow \partial_z T^{zz} = 0$



$v_\pm$ = incoming / outgoing plasma velocities (as seen from bubble wall)

$T_\pm$ = temperature outside / inside bubble

We assume planar wall (since $R_B \gg v_{EW}^{-1}$)
$\downarrow$
$(246\ \text{GeV})^{-1}$

By integrating $\partial_z T^{z0} = \partial_z T^{zz} = 0$ across bubble wall, we obtain the

<u>Matching eqs. across wall</u>

- $\omega_+ v_+^2 \gamma_+^2 + p_+ = \omega_- v_-^2 \gamma_-^2 + p_-$
- $\omega_+ v_+ \gamma_+^2 = \omega_- v_- \gamma_-^2$

→ note that the difference in vacuum energies across the bubble wall ΔV enters in $p_+ - p_- \Rightarrow$ pressure difference

→ The matching eqs. relate v_+, v_-, T_+, T_- . We'll see later how v_w, T_N (the temperature of the Universe during the EWPT) and $\alpha_N = \frac{\Delta V}{a_+ T_N^4}$ can be related to $v_\pm, T_\pm$.

We further need eq. of state for plasma.

$\longrightarrow$ $p = -F(\phi)$ (free-energy)

$\longrightarrow$ Simple approximation : "Bag e.o.s" $\begin{cases} p_+ = \frac{1}{3} a_+ T_+^4 - \underbrace{\varepsilon}_{\Delta V} \\ p_- = \frac{1}{3} a_- T_-^4 \end{cases}$

We massage the matching eqs. across wall and obtain:

$$v_+ = \frac{1}{\underbrace{1+\alpha_+}_{\alpha_+ = \frac{\Delta V}{a_+ T_+^4}}} \left[\left(\frac{v_-}{2} + \frac{1}{6 v_-} \right) \pm \sqrt{\left(\frac{v_-}{2} + \frac{1}{6 v_-} \right)^2 + (\alpha_+ + 1)(\alpha_+ - 1/3)} \right]$$

<u>+ Branch of solutions</u> ; $v_+ > v_-$; v_+^{min} for $v_-^2 = 1/3$ $(v_- = c_s)$

speed of sound of relativistic plasma

<u>- Branch of solutions</u> ; $v_- > v_+$; $\alpha_+ < 1/3$

for a sufficiently big ΔV, this branch of solutions does not exist.

② Energy-momentum conservation away from bubble wall

- Reference frame of bubble center = Reference frame of Universe.
- Away from wall, $T^{\phi}_{\mu\nu} = Ct$, and can be ignored
- We want to know the velocity profile $v(r,t)$ and temperature profile $T(r,t)$ of the plasma surrounding the bubble as the bubble expands

Spherical symmetry

$$\partial_\mu T^{\mu\nu}_{Plasma} = 0 \longrightarrow u^\nu \partial_\mu (u^\mu \omega) + u^\mu \omega \, \partial_\mu u^\nu - \partial^\nu p = 0$$

No length scale in the problem $\longrightarrow$ assume self similar solution $\to v(\xi = r/t)$, $T(\xi = r/t)$

(ansatz $\Rightarrow$ supported by numerical simulations)

$$\begin{cases} 2\dfrac{v(\xi)}{\xi} = \gamma^2(\xi)\left(1-\xi\cdot v(\xi)\right)\left[\dfrac{\mu^2(\xi)}{c_s^2} - 1\right]\partial_\xi v(\xi) \\[2ex] \left(1-\xi\cdot v(\xi)\right)\dfrac{\partial_\xi p}{\omega} = \gamma^2(\xi)\left(\xi - v(\xi)\right)\partial_\xi v \end{cases}$$

with $c_s^2 = \dfrac{(\partial p/\partial T)}{(\partial e/\partial T)} = 1/3$

$$\mu(\xi) = \frac{\xi - v(\xi)}{1-\xi\cdot v(\xi)}$$

(Lorentz-transformed fluid velocity)

for fixed t, just $v(r)$, $T(r)$

- We can compute $v(\xi)$ and $T(\xi)$ as the bubble expands by solving these two eqs with the boundary conditions on the bubble wall (given by the matching eqs across the wall ①)

- These eqs only accept solutions of a certain kind, which in turn allows to relate $v_\pm$, $T_\pm$ with V_w, T_N for each type of solution.

① + ②

Two sets of solutions : <u>Bubble expansion modes</u>

- No fluid motion in front of bubble wall: $v(\xi > \xi_w) = 0$

$\rightarrow$ $\underline{V_w = \xi_w = v_+ > c_s}$ (Supersonic solutions)

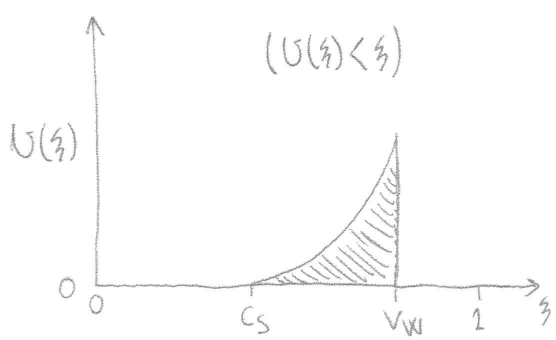
$\rightarrow$ $v_+ > v_-$

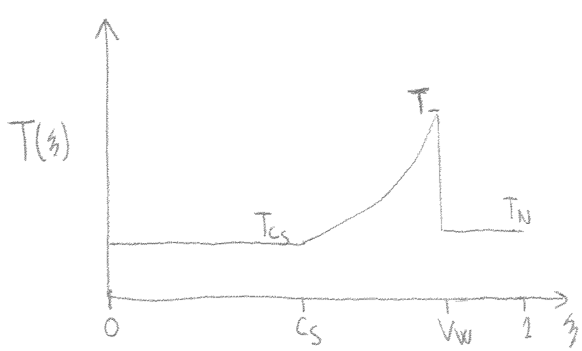
$\rightarrow$ $v(V_w) = \mu(V_w, v_-) \longrightarrow v_- = \mu(V_w, v(V_w))$

Since $\partial_\xi v > 0$, $\mu(\xi) > c_s \Longleftrightarrow \mu(\xi) > \mu(V_w) = v_- \geq c_s$

$\rightarrow$ $T_+ = T_N$; $T_- > T_N$

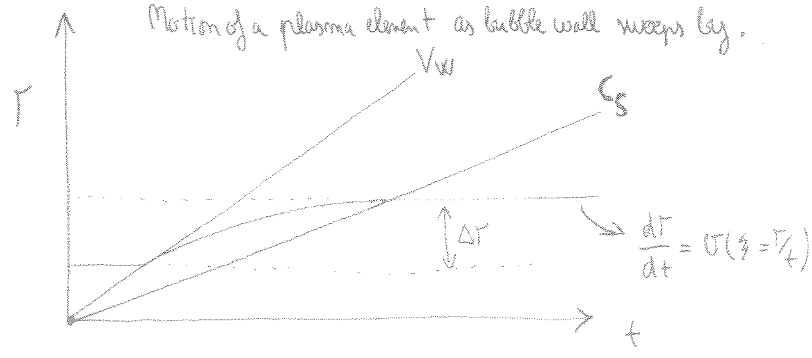
DETONATION SOLUTIONS





No plasma motion in front of wall: a particle just outside the wall sees the EWPT front (bubble wall) approaching with speed V_w. Once wall passes, the plasma behind is set into motion (rarefaction wave) and heated up.

Motion of a plasma element as bubble wall sweeps by.



- No fluid motion behind wall : $v(\xi < \xi_w) = 0$

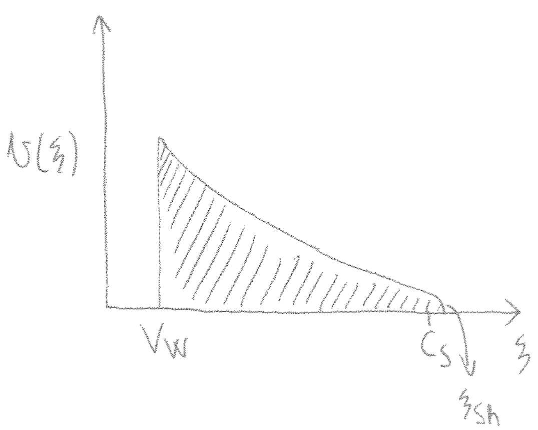
$\rightarrow$ $\underline{V_w = \xi_w = v_- > v_+}$ ($\underline{\text{Subsonic solutions}}$)

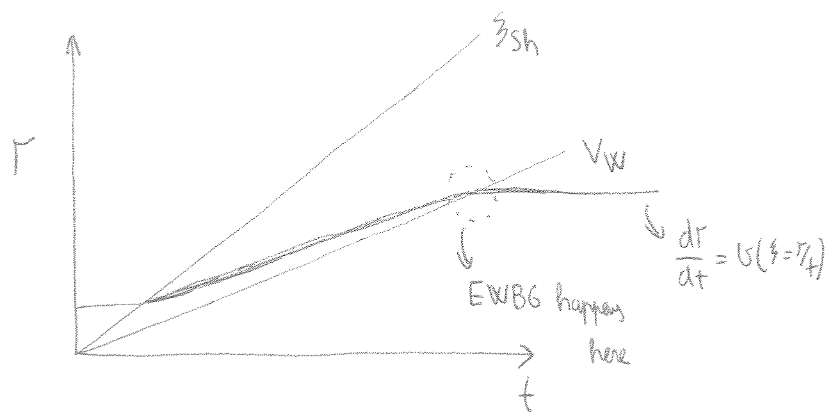
$\rightarrow$ $T_+ > T_N$; $T_- < T_+$

$\rightarrow$ $\mu(V_w, v(V_w)) < c_s \rightarrow \partial_\xi v < 0$

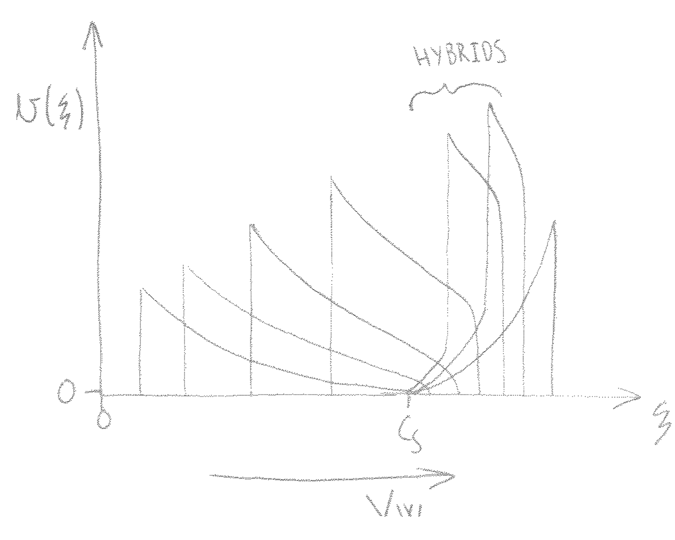
DEFLAGRATION SOLUTIONS

The slowly expanding bubble pushes the plasma in front and sets it into motion (deflagration front) and heats it up.





Note that as we increase V_w, we go from deflagrations to detonations. This change is not abrupt, but is smooth (transition between the two solutions happens via "HYBRID SOLUTIONS")



Impact

See Chiara's talk!

- **GW:** Ω_{GW} strongly depends on v_w and α_N via $K(\alpha_N, v_w)$

$K \equiv$ available energy for GW generation (energy in bulk motion) normalized to the vacuum energy of transition.

$$K(\alpha_N, v_w) = \frac{3}{\Delta V\, v_w^3} \int \omega(\xi)\, v^2(\xi)\, \gamma^2(\xi)\, \xi^2\, d\xi$$

$$= 3 \cdot \frac{\int_0^{R_B} T^{rr}(\omega(\xi), v(\xi))\, r^2\, dr}{\Delta V \cdot R_B^3}$$

- **EWBG:**

If $v_w < c_s$ (subsonic; deflagrations), diffusion/transport in EWBG works fine

- Because of hydrodynamics, $v_w > v_+$

This is the velocity relevant for transport, since is the velocity of bubble wall relative to plasma in front.

$\Downarrow$

Can significantly affect (and improve) transport, specially for $v_w \to c_s$

No (1103.2159)

If supersonic (detonations), diffusion/transport in EWBG <u>does not work</u>
$v_w > c_s$

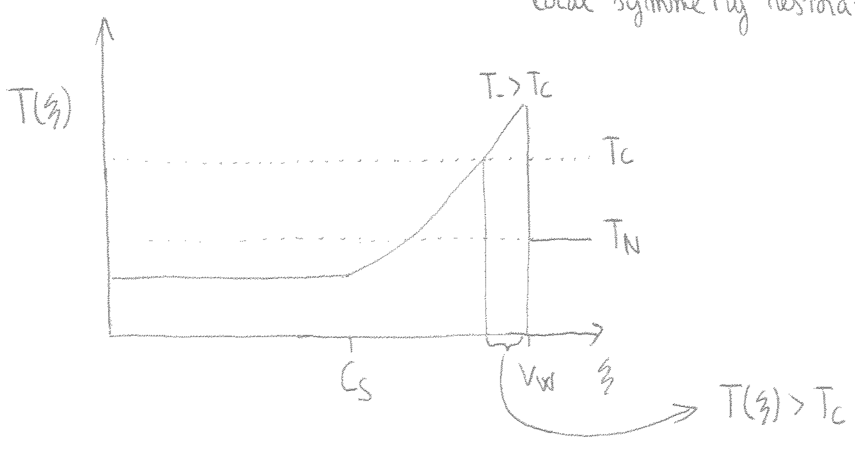
$\downarrow$

Standard EWBG does not work

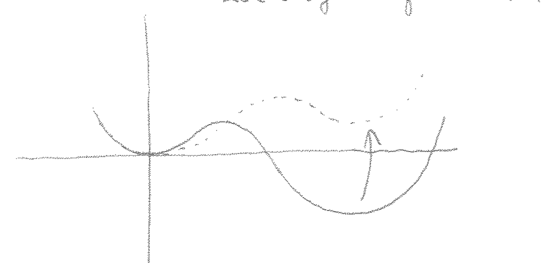
However, detonation properties can still allow for baryogenesis:
Caprini, No (1111.1726)

$T_- > T_+ = T_N \implies$ Heating-up behind bubble wall

If heating-up such that $T(\xi \to v_w^-) > T_c$,
local symmetry restoration?



"Local symmetry restoration"



Symmetric bubble nucleation can occur $\implies$ Sphalerons active inside "symmetric" bubbles and EWBG can occur, even when "mother bubble is supersonic

The mechanism for baryogenesis is the usual one, but takes place in the symmetric bubbles, whose expansion velocity w.r.t. the plasma around is

$$\delta v(\xi) \simeq \frac{T(\xi)}{T_c} - 1 \ll c_s$$